Self Inductance Formulas for Multi-Turn Rectangular Loops Used with Vehicle Detectors

Milton K. Mills
U.S. Department of Transportation
Federal Highway Administration
Office of Safety and Traffic Operations R&D
Systems Technology Division
Washington, D.C. 20590

Introduction

This paper describes the development and practical application of a formula to calculate the self inductance of a multi-turn, rectangular loop of round wire. The multi-turn loop is modeled as a number of series connected, single turn loops which exhibit mutual coupling between each other. Inductive loop detectors use a multi-turn, rectangular loop which is buried in the roadway. When a vehicle passes over the loop, the resulting inductance change is coupled to roadside detector electronics by a transmission line.

A number of formulas exist in the literature to accurately calculate the self inductance of a single turn, rectangular loop of round wire. The paper presents a tutorial description of the theory for some of these formulas. Formulas in the literature for calculating the self inductance of multi-turn rectangular loops are also discussed.

Background

Although a number of formulas exist in the literature to calculate the self inductance of a multi-turn, rectangular loop in free space, the more accurate formulas which use an equivalent current sheet either involve a number of terms or require tables. As a result simple "rule of thumb" or empirical formulas have been used to calculate tables of loop self inductance values versus loop size. Unfortunately, many of the calculated values are erroneous because the simple formulas have been used outside their range of validity which is normally not specified.

Self Inductance by Selected Mutual Inductance

The external self inductance of a loop conductor with finite cross sectional area is obtained by dividing the external flux normal to the enclosed loop area by the current in the loop. If the current distribution in the loop conductor is uniform (i.e., not disturbed by other portions of the loop), the loop self inductance[2] is found from the mutual inductance between a line current in the center[3] of the conductor and a line circuit along the surface of the conductor.

Mutual Inductance of Parallel Filamentary Circuits

The mutual inductance[4] between the pair of filamentary circuits located in free space and illustrated in Figure 1 is:

$$M(l,d) = \pm \frac{\mu_o l}{2\pi}\left\{\ln\left[\frac{l}{d} + \sqrt{1 + \left(\frac{l}{d}\right)^2}\right] - 1 \pm \sqrt{\left(\frac{d}{l}\right)^2} + \frac{d}{l}\right\} \qquad (1)$$

where

μ_o = permeability of free space = $4\pi \times 10^{-9}$ Hy/m
l = filamentary length (m)
d = filamentary spacing (m)

M (l,d) = mutual inductance (Hy)

A plus sign is used when the direction of current in the filaments is the same and a minus sign is used when the direction of current in the filaments is opposite. When the filament length is much greater than the filament spacing (i.e., l>>d which applies to transmission lines), then:

$$M(l,d) = \pm \frac{\mu_o l}{2\pi}\left[\ln\left(\frac{2l}{d}\right) - 1\right]$$

where

l>>d (2)

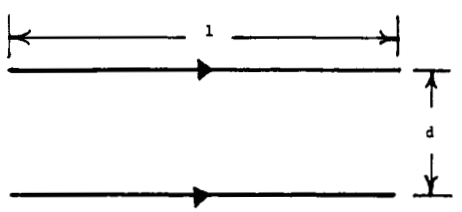

Fig. 1. Pair of Parallel Current Elements

External Self Inductance of Single Conductor

The external self inductance of the single conductor in Figure 2 is determined using the mutual inductance method. The conductor is replaced by two, parallel filamentary circuits with a spacing equal to the conductor radius, ρ. Letting d equal ρ in equation (1), the external self inductance is:

$$L^e = \frac{\mu_o l}{2\pi}\left\{\ln\left[\frac{l}{d} + \sqrt{1 + \left(\frac{l}{d}\right)^2}\right] - \sqrt{1 + \left(\frac{d}{l}\right)^2} + \frac{d}{l}\right\} \qquad (3)$$

When the conductor is long so that the end effect is negligible,

$$L^e = \frac{\mu_o l}{2\pi}\left[\ln\left(\frac{2l}{\rho}\right) - 1\right] \qquad (4)$$

where $l >> \rho$

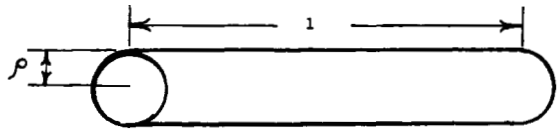

Fig. 2. Single Conductor

External Self Inductance of Conductor Pair

The external self inductance of the conductor pair in Figure 3 is given by

$$L_p = L_1 - M_{12} + L_2 - M_{21} \qquad (5)$$

The mutual inductance is minus because of the opposing currents in the two conductors. Since the two conductors have the same radius, ρ , and length, l,

$$L = L_1 = L_2 \text{ and } M = M_{12} = M_{21}$$

since the mutual inductance is symmetrical.

Then

$$L_p = 2\ (L - M) \qquad (6)$$

Substituting equations (1) and (3) into equation (6), the external self inductance of the conductor pair including end effect is:

$$L^e_p = \frac{\mu_o l}{\pi}\left\{\ln\left[\frac{l}{\rho} + \sqrt{1 + \left(\frac{l}{\rho}\right)^2}\right] - \sqrt{1 + \left(\frac{\rho}{l}\right)^2} + \frac{\rho}{l} - \ln\left[\frac{l}{d} + \sqrt{1 + \left(\frac{l}{d}\right)^2}\right] + \sqrt{1 + \left(\frac{d}{l}\right)^2} - \frac{d}{l}\right\} \qquad (7)$$

If the conductor pair is embedded in a medium of relative permeability, μ_r, μ_o in equation (7) is multiplied by μ_r. If the length of the conductors is much greater than the conductor spacing and conductor diameter

$$L^e_p = \frac{\mu_o l}{\pi}\ln\left(\frac{d}{\rho}\right) \qquad (8)$$

where

$l >> d$

$l >> \rho$

This equation can be used to calculate the inductance of a parallel two wire transmission line.

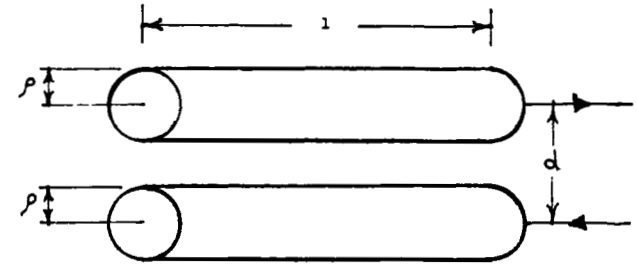

Fig. 3 Conductor Pair

External Inductance of Single Turn Rectangular Loop

The external inductance of a single turn, rectangular loop is given by the sum of the inductance of two pairs of conductors. Then:

$$L^e_o = L^e_{p1} + L^e_{p2} \qquad (9)$$

Substituting equation (7) into equation (9) and collecting terms, the external inductance of a single turn, rectangular loop is:

$$L^e_o = \frac{\mu_o}{\pi}\left\{ l_1 \ln\left[\frac{l_1}{\rho} + \sqrt{1 + \left(\frac{l_1}{\rho}\right)^2}\right] - l_1 \ln\left[\frac{l_1}{l_2} + \sqrt{1 + \left(\frac{l_1}{l_2}\right)^2}\right] + l_2 \ln\left[\frac{l_2}{\rho} + \sqrt{1 + \left(\frac{l_2}{\rho}\right)^2}\right] - l_2 \ln\left[\frac{l_2}{l_1} + \sqrt{1 + \left(\frac{l_2}{l_1}\right)^2}\right] - \sqrt{l_1^2 + \rho^2} - \sqrt{l_2^2 + \rho^2} + 2\sqrt{l_1^2 + l_2^2} - (l_1 + l_2) + 2\rho \right\} \qquad (10)$$

where

l_1 = width of loop (m)

l_2 = length of loop (m)

This equation can be written in more compact form by combining logarithms.

Internal Inductance Per Unit Length of a Cylindrical Conductor

Johnson[5] shows that the ratio of actual internal inductance to low-frequency internal inductance is given by:

$$\frac{L_i}{L_{io}} = \frac{4}{q}\left[\frac{\mathrm{bei}\,q\,\mathrm{bei}'q + \mathrm{ber}\,q\,\mathrm{ber}'q}{(\mathrm{bei}'q)^2 + (\mathrm{ber}'q)^2}\right] \quad (11)$$

where

$$q = \frac{\rho\sqrt{2}}{\delta}$$

where

$$\delta = \frac{1}{\sqrt{\mu_r \mu_o \pi f \sigma}} \quad (12)$$

L_i = internal inductance (Hy/m)
δ = skin depth (m)
μ_o = permeability of free space = $4\pi \times 10^{-7}$ Hy/m
μ_r = relative permeability of copper wire = 1
f = frequency (Hz)
σ = conductivity of copper wire = 0.58×10^{8} (mhos/m)
ρ = radius of wire (m)

$$L_{io} = \frac{\mu_o \mu_r}{8\pi} \text{ (Hy/m)} \quad (13)$$

for copper wire

$$L_{io} = 0.5 \times 10^{-7} \text{ (Hy/m)} \quad (14)$$

Equations (11) and (12) were programmed on a Hewlett Packard 97 programmable calculator and the resulting internal inductance versus frequency for numbers 12, 14, and 16 AWG copper wire is presented in Table 1. A value of 0.036 micro Henries per meter for number 14 AWG copper wire at 47 KiloHertz was used for all loop inductance calculations.

Self Inductance of Single Turn Rectangular Loop

The self inductance of a single turn rectangular loop is given by the sum of internal and external inductance and is:

$$L_o = L^i_o + L^e_o \quad (15)$$

where

$$L^i_o = 2(l_1 + l_2)L^i \quad (16)$$

and L^i is given by equation (11) and L^e_o is given by equation (10).

Table I

Internal Inductance versus Frequency for Copper Wire

American Wire Gauge No:	12	14	16
Frequency (KHz)	Internal Inductance (μH/m)	Internal Inductance (μH/m)	Internal Inductance (μH/m)
1	0.0500	0.0500	0.0500
5	0.0493	0.0497	0.0499
10	0.0473	0.0489	0.0495
15	0.0445	0.0475	0.0490
20	0.0414	0.0459	0.0482
25	0.0385	0.0440	0.0473
30	0.0358	0.0421	0.0463
35	0.0351	0.0402	0.0452
40	0.0315	0.0384	0.0440
45	0.0298	0.0367	0.0428
47	0.0216	0.0361	0.0423
50	0.0283	0.0351	0.0416
55	0.0270	0.0337	0.0404
60	0.0259	0.0324	0.0392
65	0.0249	0.0312	0.0381
70	0.0240	0.0301	0.0371
75	0.0232	0.0291	0.0360
80	0.0225	0.0282	0.0351
85	0.0218	0.0274	0.0341
90	0.0212	0.0266	0.0333
95	0.0207	0.0259	0.0325
100	0.0202	0.0253	0.0317
105	0.0197	0.0247	0.0310
110	0.0193	0.0241	0.0303

Simplified Formula for Single Turn Rectangular Loop

From a combination of theoretical and experimental results, LINK[6] developed the formula:

$$L\ (\mu H) = 0.5P \qquad (17)$$

where

P = loop perimeter (ft)

An accuracy of 10 percent is specified for the formula for typical loop sizes.

In Appendix I, the single turn loop is approximated by two series connected transmission lines which neglect the end effect terms in the more exact formula. The inductance formula for a single turn loop of number 14 gauge wire is:

$$L\ (\mu H) = 0.45P \qquad (18)$$

The size of the coefficient is a function of the wire gauge. The coefficient for number 20 gauge wire is 0.49.

Multi-Turn Loop Inductance Formulas

Three formulas are used to calculate the inductance of a multi-turn, rectangular loop. The first formula called the coupling coefficient formula, uses a coupling coefficient to account for leakage flux. The coupling coefficient is determined experimentally from loop inductance measurements. The second formula called the current sheet formula, models the inductor winding with an equivalent current sheet. The third formula called the mutual coupling formula, models the inductor winding as a number of single turns connected in series and mutually coupled to each other.

Multi-Turn, Rectangular Loop Inductance Formula Using Coupling Coefficient

The calculation of the inductance of a multi-turn, rectangular loop using a coupling coefficient is described by LINK[7] and the formula is:

$$L = 0.01016\Big\{\mu_w \delta' N(S_1+S_2) + \mu_m (KN)^2 \Big[2\left(g+\frac{d}{2}\right) - 2(S_1+S_2) + (S_1+S_2)\ln\frac{4S_1S_2}{d} - S_1\ln(S_1+g) - S_2\ln(S_2+g)\Big]\Big\} \qquad (19)$$

where

L = loop inductance (μH)
μ_w = relative permeability of copper wire=1
δ' = skin effect factor = 0.18
S_1 = short side of rectangle (in)
S_2 = long side of rectangle (in)
d = wire diameter (in)

$$g = \sqrt{S_1^2 + S_2^2}$$

μ_m = relative permeability of medium = 1

K = coupling coefficient = 0.87 (6 ft x 6 ft loop), 0.95 (6 ft x 50 ft loop), 0.94 (6 ft x 30 ft loop)

A simplified formula developed by LINK[8] is:

$$L = 0.41\ PN^2 \qquad (20)$$

where

L = loop inductance (μH)
P = perimeter (ft)
N = loop turns

LINK[9] notes that a "rule of thumb" formula is:

$$L = 0.46\ PN^2 \qquad (21)$$

Multi-Turn, Rectangular Loop Inductance Formula Using Current Sheet

The general inductance formula[10] using a rectangular current sheet was first derived by Niwa[11] in 1924 and for the coil geometry in Figure 4 is:

$$\begin{aligned} L(\mu H) = 0.008N^2\frac{aa_1}{b}\Big[&\frac{1}{2}\frac{b}{a_1}\sinh^{-1}\frac{a}{b} + \frac{1}{2}\frac{b}{a}\sinh^{-1}\frac{a_1}{b} \\ &- \frac{1}{2}\left(1-\frac{a_1^2}{b_1^2}\right)\frac{b}{a_1}\sinh^{-1}\frac{a}{b\sqrt{1+\frac{a_1^2}{b^2}}} \\ &- \frac{1}{2}\left(1-\frac{a^2}{b^2}\right)\frac{b}{a}\sinh^{-1}\frac{a_1}{b\sqrt{1+\frac{a^2}{b^2}}} \\ &- \frac{1}{2}\frac{a_1}{b}\sinh^{-1}\frac{a}{a_1} \\ &- \frac{1}{2}\frac{a}{b}\sinh^{-1}\frac{a_1}{a} + \frac{\pi}{2} - \tan^{-1}\frac{aa_1}{b^2\sqrt{1+\frac{g^2}{b^2}}} \\ &+ \frac{1}{3}\frac{b^2}{aa_1}\sqrt{1+\frac{g^2}{b^2}}\left(1-\frac{1}{2}\frac{g^2}{b^2}\right) \\ &- \frac{1}{3}\frac{b^2}{aa_1}\sqrt{1+\frac{a^2}{b^2}}\left(1-\frac{1}{2}\frac{a^2}{b^2}\right) \\ &- \frac{1}{3}\frac{b^2}{aa_1}\sqrt{1+\frac{a_1^2}{b^2}}\left(1-\frac{1}{2}\frac{a_1^2}{b^2}\right) \\ &+ \frac{1}{6}\frac{b}{aa_1}\left(\frac{g^3-a^3-a_1^3}{b^2}\right)\Big] \end{aligned} \qquad (22)$$

where

$g^2 = a^2+a_1^2$, b = Np
N = coil turns
a_1 = short side of rectangle (cm)
a = long side of rectangle (cm)
b = axial length of coil (cm)
p = pitch (cm/turn)

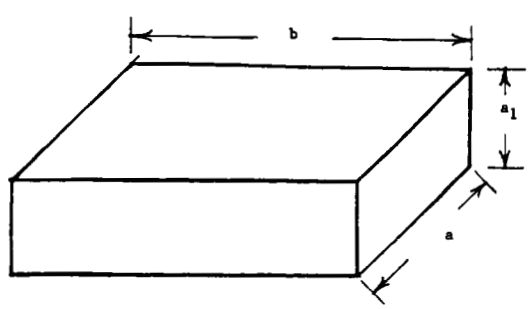

Fig. 4. Coil Geometry

Table II

Niwa's Coefficients for Short Rectangular Solenoid

K	β_1	β_1'	β_2	β_3	β_5	β_7
1.00	0.4622	0.6366	0.2122	-0.0046	0.0046	-0.0382
0.95	0.4574	0.6534	0.2234	-0.0046	0.0053	
0.90	0.4512	0.6720	0.2358	-0.0046	0.0064	-0.0525
0.85	0.4448	0.6928	0.2496	-0.0042	0.0080	
0.80	0.4364	0.7162	0.2653	-0.0031	0.0103	-0.0838
0.75	0.4260	0.7427	0.2829	-0.0010	0.0141	
0.70	0.4132	0.7730	0.3032	0.0026	0.0198	-0.1564
0.65	0.3971	0.8080	0.3265	0.0085	0.0291	
0.60	0.3767	0.8488	0.3537	0.0179	0.0432	-0.3372
0.55	0.3500	0.8970	0.3858	0.0331	0.0711	
0.50	0.3151	0.9549	0.4244	0.0578	0.1183	-0.7855
0.40	0.1836	1.1141	0.5305	0.1697	0.3898	-2.403
0.30	-0.0314	1.3359	0.7074	0.5433	2.0517	-7.85
0.20	-0.6409	1.9099	1.0610	2.3230	14.507	15.51
0.10	-3.2309	3.5014	2.1220	22.548	497.36	14280

Grover[12] states that a disadvantage of this formula is the near cancellation of terms of opposite signs in some cases. For a very short solenoid, Grover provides Niwa's formula for an infinite solenoid modified by a factor F'. The formula[13] is:

$$Lu(\mu H) = 0.004\pi N^2 \frac{a a_1}{b} F' \qquad (23)$$

where

$$F' = \beta_1 \gamma + \beta_1' \gamma \ln\frac{1}{\gamma} + \beta_2 \gamma^2 + \beta_3 \gamma^3 - \beta_5 \gamma^5 + \cdots$$

and $\gamma = \frac{b}{a} = \frac{\text{length of current sheet}}{\text{longer side of rectangle}}$

$$K = \frac{a_1}{a}$$

The value of B_1' and B_1 is obtained from Table[14] II.

Multi-Turn, Rectangular Loop Inductance Formula Using Mutual Coupling

The mutual coupling formula is used to calculate the inductance of a multi-turn, rectangular loop. The formula was used by Niwa[15] to calculate the inductance of a single layer coil with concentrated windings at equal intervals. King[16] determined the impedance of a helical coil using this formula.

A three turn, rectangular loop is modeled as three single turn loops connected in series as shown in Figure 5. A circuit model of the three turn loop is illustrated in Figure 6. The total inductance is:

$$L_T = L_1+L_2+L_3+M_{12}+M_{13}+M_{21} +M_{23}+M_{31}+M_{32}$$

Since $L_1=L_2=L_3=Lo$ and for equal turn spacing $M_{12}=M_{21}=M_{23}=M_{32}$

$M_{13}=M_{31}$

$$L_T=3Lo+4M_{12}+2M_{13} \qquad (24)$$

If the three coil turns are wrapped around a high permeability toriod, the mutual inductance is approximately equal to the inductance of a single turn giving:

$$L_T = 9\ Lo = N^2 Lo \qquad (25)$$

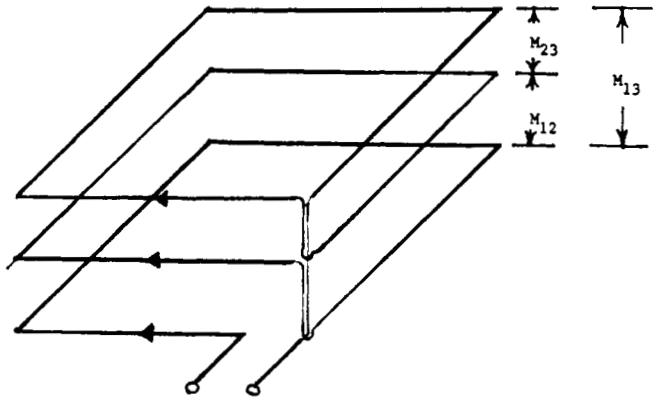

Fig. 5 Three Turn Inductive Loop Model

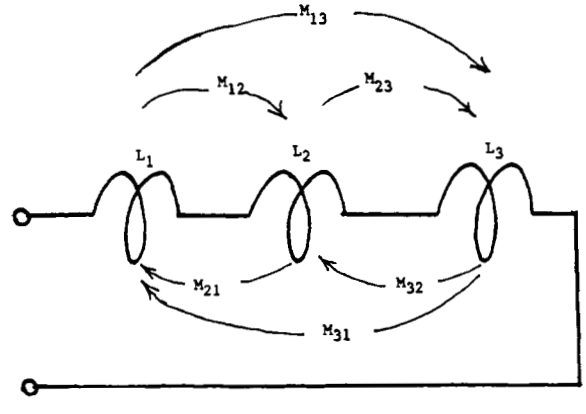

Fig. 6 Circuit Model of Three Turn Loop

For air coupled coils, the mutual inductance is less than the single turn inductance because of leakage flux.

The general inductance formula for a coil with N equal spaced, identical turns is:

$$L_T = NLo + 2(N-1)M_{12} + 2(N-2)M_{13} + \ldots \quad (26)$$

Mutual Inductance of Two Coaxial, Parallel, Rectangular Loops

The total mutual inductance of the rectangular loops in Figure 7 is given by the sum of the mutual inductances between the parallel sides and using formula (1) is:

$$M = 2\left[M_{13}(A, \sqrt{H^2+B^2}) - M_{11}(A,H) + M_{24}(B, \sqrt{H^2+A^2}) - M_{22}(B,H)\right] \quad (27)$$

where M_{11} is the mutual inductance between side 1 of the bottom loop turn and side 1 of the top loop turn under consideration as shown. Note that all mutual inductances are symmetrical (i.e., $M_{13}, = M_{31}$, etc.).

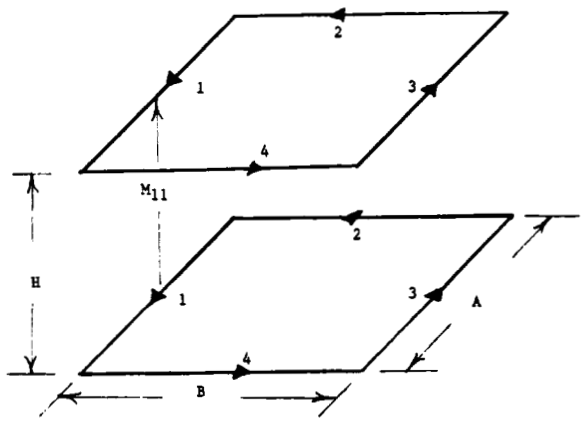

Fig. 7 Loop Turn Geometry

Calculator Self Inductance Program

Serveral programs were developed for use on the Hewlett Packard 97 programmable calculator. One program calculates the self inductance of each single turn loop using equations (10), (15), and (16). Another program calculates the mutual inductance of two parallel, coaxial, rectangular loops using equation (27). The results of the two programs are applied to equation (26) to calculate the self inductance of a multi-turn rectangular loop.

Comparison of Calculated Loop Self Inductance Using Multi-Turn, Rectangular Loop Inductance Formulas

The self inductance of a 1.83m (6 ft.) by 1.83m (6 ft.), three turn loop is calculated in Appendix II assuming number 14 gauge wire and a conductor spacing of 0.38cm (150 mils). The self inductance values are:

Formula	Loop Self Inductance (μH)
LINK Coupling Coefficient (Eqn 19)	76.9
LINK Simplified (Eqn 20)	88.6
Niwa Current Sheet (Eqn 23)	75.8
Mutual Coupling (Eqn 26)	75.6

Comparison of Calculated and Measured Loop Self Inductance

Table III presents a comparison of calculated loop self inductance values by the mutual coupling formula to measured loop self inductance values. Unfortunately, the conductor spacing for the loops used for measurements is unknown. A value 0.38cm (150 mils) was assumed for all theoretical calculations. The inductance values obtained by the mutual coupling formula are reasonably accurate as seen by the favorable comparison with a range of measured loop inductances. The effect of loop capacitance[17] between turns was neglected.

Results and Conclusions

The loop self inductance values in Table IV were calculated using the mutual coupling formula, equation (26). The loop capacitance between turns was neglected. In practice, loops larger than 1.83m (6 ft) by 10.67m (35 ft) normally use a maximum of two turns.

The effect of conductor spacing on loop self inductance is easily determined using the mutual coupling formula. For example, a seven turn, 1.83m (6 ft) by 1.83m (6 ft) loop with 0.38cm (150 mil) conductor spacing has a self inductance of 355 microHenries. If the conductor spacing is increased to 0.5cm (200 mil), the self inductance is 339 microHenries.

The mutual coupling formula, equation (26), provides acceptable accuracy for calculating the self inductance of multi-turn, rectangular loops which have a large area relative to the conductor spacing. This type of loop is used in vehicle detector systems. Since programmable calculators are widely available, the mutual coupling formula is recommended rather than less accurate "rule of thumb" formulas.

Table III

COMPARISON OF CALCULATED AND MEASURED LOOP INDUCTANCES

Number of Loop Turns	Loop Size 5 ft x 5 ft		Loop Size 6 ft x 6 ft		Loop Size 6 ft x 30 ft		Loop Size 6 ft x 50 ft	
	Calculated Inductance (μH)	Measured Inductance* (μH)	Calculated Inductance (μH)	Measured Inductance (μH)	Calculated Inductance (μH)	Measured Inductance (μH)	Calculated Inductance (μH)	Measured Inductance (μH)
1			10.4	15	34	35	53	50
2			36.6	35	126***	125	205	200
3	67.7	65	75.6	75	272	275	447	430
4			129.2	135	461	475	773	750
5	161	165	193.6	205				
6								
7			355	365**				

* R. W. Lyles, J. H. Wyman, "Evaluation of Speed Monitoring Systems," Report No. FHWA/PL/80/006, July 1980.
** W. A. Blikken, "The Adjustable Loop for Vehicle Detection," Freeway Operations Unit, Michigan DOT.
*** The loop inductance is 118 μH using the current sheet formula.

NOTE: Other measured values from LINK, Advanced Products Division, "Link Inductance Loop Analysis Project, Final Report," August 1968.

Table IV

CALCULATED LOOP INDUCTANCE

Loop Size (Feet)	Loop Inductance (Microhenries)						
	1 Turn	2 Turn	3 Turn	4 Turn	5 Turn	6 Turn	7 Turn
5x5	9	30	62	104	155		
6x6	10	37	76	129	194	269	355
6x10	14	51	107	181			
6x15	19	69	147	249			
6x20	24	88	187	320			
6x22	26	96	204	349			
6x25	29	107	229	392			
6x30	34	126	272	461			
6x35	39	146	315	542			
6x40	43	165	359	618			
6x45	48	185	402	695			
6x50	53	205	447	773			
6x55	58	225	492	853			
6x60	63	245	537	932			
6x65	67	265	583	1012			
6x70	72	286	628	1092			

Spacing between loop conductors: 150 MIL center to center

APPENDIX I

By approximating the single turn loop by two series connected transmission lines, the loop inductance formula is:

$$L_o(\mu H) = L_{pair2} + L_{pair2}$$

The low frequency inductance of a twin lead, transmission line comprised of copper wire in free space is:

$$L_{pair1} = \left[10^{-1} + 0.4 \ln \left(\frac{W}{\rho}\right)\right] L$$

Where

L_{pair1} = inductance of pair 1 (μH)
W = transmission line conductor spacing (m)
L = transmission line length (m)
ρ = radius of conductor (m)

Similarly

$$L_{pair2} = \left[10^{-1} + 0.4 \ln \left(\frac{L}{\rho}\right)\right] W$$

Then

$$L_o(\mu H) = \left[10^{-1} + 0.4 \ln \left(\frac{W}{\rho}\right)\right] L + \left[10^{-1} + 0.4 \ln \left(\frac{L}{\rho}\right)\right] W$$

$$L_o(\mu H) = \left[10^{-1} + 0.4 (\ln W - \ln \rho)\right] L + \left[10^{-1} + 0.4 (\ln L - \ln \rho)\right] W$$

If $-\ln \rho > \ln W$ and $-\ln \rho > \ln L$

$$L_o(\mu H) = (10^{-1} - 0.4 \ln \rho)(L+W)$$

If $P = 2L+2W$

Then $L_o(\mu H) = (0.05 - 0.2 \ln \rho) P$

Where P = loop perimenter (m)

From Table I-1, $= 0.814 \times 10^{-3}$m for numbers 14 AWG wire. Then

$$L_o\ (\mu H) = 0.45\ P(ft)$$

Table I-1

Conductor Size

AWG	Wire Dia (Mils)	Wire Dia (mm)	Wire Radius (mm)
12	80.8	2.052	1.026
14	64.1	1.628	0.814
16	50.8	1.290	0.645
18	40.3	1.024	0.512
20	32.0	0.813	0.407

Appendix II

Calculation of Self Inductance of Three Turn, Six Foot by Six Foot Loop Using Coupling Coefficient Formula

Using equation 19, the inductance is given by:

$$L = 0.01016 \Big\{ (1)(0.18)(3)(72+72) + 1\left[(0.87)(3)\right]^2 2\left(\sqrt{(72)^2+(72)^2} + 32.05\times10^{-3}/2\right) - 2(72+72) + (72+72)\ln\left[4(72)(72)/32.05\times10^{3}\right] - 72\ln\left(72 + \sqrt{(72)^2+(72)^2}\right) - 72 \ln\left(72 + \sqrt{(72)^2+(72)^2}\right) \Big\}$$

$$L = 0.01016 \left\{ 77.76 + 6.812\left[203.65 + 32.05\times10^{-3} - 288 + 1926.73 - 371.38 - 371.38\right]\right\}$$

$$L = 76.9\ \mu H$$

Calculation of Self Inductance of Three Turn, Six Foot by Six Foot Loop Using Current Sheet Formula

al = 6ft = 182.93 cm
a = 6ft = 182.93 cm

P = turn spacing = 150 mils = 0.381 cm

$N_p = 3(0.381) = 1.143$ cm

$$\gamma = \frac{b}{a} = \frac{1.143}{182.93} = 0.0062$$

$$K = \frac{a_1}{a} = 1$$

From Table II,

$B_1 = 0.4622$

$B_1' = 0.6366$

$B_2 = 0.2122$

$B_3 = 0.0046$

$B_5 = 0.0046$

$B_7 = 0.0382$

$\ln \frac{1}{\gamma} = 5.0754$

$$F' = B_1\gamma + B_1'\gamma \ln\frac{1}{\gamma} + B_2\gamma^2 + B_3\gamma^3 - B_5\gamma^5 + \ldots$$

$$F' = 0.0029 + 0.0200 + 8.16\times10^{-6}$$

$$F' = 0.0229$$

Using equation (23)

$$L = 0.004\pi N^2 \frac{aa_1}{b} F^1$$

$$L = 0.004\pi(9)\frac{(182.93)(182.93)}{1.143}(0.0229)$$

$$L = 75.8\ \mu H$$

Calculation of Self Inductance of Three Turn, Six Foot by Six Foot Loop Using Mutual Coupling Formula

From Table I and equation 16

$Lo^i = (0.036 \mu H/m)(2)(l_1+l_2)$

$Lo^i = (0.036 \mu H/m)(2)(1.829+1.829)$

$Lo^i = .26 \mu H$

Using equation (10)

$Lo^e = 10.16 \mu H$

Then

$Lo = Lo^i + Lo^e$

$Lo = 0.26 \mu H + 10.16 \mu H = 10.4 \mu H$

Using equation (26)

$L = 3\ Lo + 4\ M_{12} + 2\ M_{13}$

From equation (27) for 150 mil spacing between turns

$M_{12} = 7.9 \mu H$

$M_{13} = 6.9 \mu H$

Then

$L = 3(10.4 \mu H) + 4(7.9 \mu H) + 2(6.9 \mu H)$

$L = 75.6 \mu H$

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