PERMITTIVITY TENSOR AND RF DISSIPATION IN PLASMAS OF LOW ASPECT RATIO TOROIDAL DEVICES.

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Introduction

Recently, several experiments in spherical or low aspect ratio tokamaks (LART) (where share of trapped particles is very large) have produced efficient plasma confinement in high beta regimes and, nowadays, the radio frequency (RF) plasma heating and current drive studies are very important parts of experimental and theoretical programs on LART (see, for example, NSTX program [1] where fast wave heating and current drive is proposed). In LART, electrons moving along magnetic field suffer strong velocity modulation (bouncing effect) that can affect RF dissipation. The analysis of wave heating and current drive by RF fields can be carried out on the basis of Vlasov-Maxwell set of equations. For a LART model with concentric circular magnetic surfaces, the Vlasov's equation was solved analytically (see for example Ref.[2]) and parallel dielectric tensor was found via Jacobi functions. Finally, numerical calculations of imaginary part of the parallel tensor component [4],[5] had demonstrated that bounce effect can strongly affect RF dissipation in the LART. Here, using the same procedure as in Ref.[2], the full set of the permittivity tensor components for solving Maxwell equations in LART is presented.

Plasma model and Vlasov equation.

In magnetized plasmas, $\rho_\lambda \langle\langle L$ (where $\rho_\lambda = \frac{v_T}{\Omega_B}$ is a Larmor radius and v_T, Ω_B, and L are thermal velocities, cyclotron frequency and characteristic space-scales, respectively) the Vlasov equation can be simplified sufficiently with approximate integrating of particle trajectory equations. In this case, we get so called gyro-kinetic equation. We use approximation so called drift equation, $\frac{\rho_\lambda}{L}\langle\langle 1$ and $\frac{\omega}{\Omega_B}\langle\langle 1$. Here, in contrast to those simplification methods, we use a traditional method (like in Chew G.F, Goldberger L.G, Low F.E hydrodynamics) of extracting of the space-magnetized plasma parameter in the Vlasov equation directly. To perform this procedure, we introduce new 6-dimensional space $\vec{r}$, $\vec{v}$ where $\vec{r}' = \vec{r}$, $v_1 = (\vec{n}\cdot\vec{u})$, $v_2 = (\vec{\tau}\cdot\vec{u})$ and $v_3 = (\vec{h}\cdot\vec{u})$ are the velocity projections on unit vectors related to the geometry of the stationary magnetic field $\vec{B}_0$. The normal and parallel vectors $\vec{n}$, $\vec{h}$ are directed along the normal to magnetic surface and along the magnetic field, respectively, and the vector $\vec{\tau}$ is defined by the equation $\vec{\tau} = \vec{h}\times\vec{n}$. Then, we introduce spherical coordinates (v, σ, γ) instead of the Cartesian coordinates $v_1 = v\, sin\gamma \cos\sigma$, $v_2 = v\, sin\gamma\, sin\sigma$, $v_3 = v\cos\gamma$ that are defined in the regions: $0 \le v \le \infty$, $0 \le \gamma \le \pi$, $0 \le \sigma \le 2\pi$.

Further, the linearized Vlasov equation (where $\partial/\partial t + \nu = -i\omega + \nu = -i\Omega$ is used because of the Krook form of the collision term $St[f] = -\nu f$) can be written for Fourier components f_l of the distribution function $f = \sum f_l \, e^{il\sigma}$ in the form:

$$\left(\hat{D}_0 - i\Omega\right)f_0 + \left(\hat{D}^+ + i\omega^+\right)f_1 + \left(\hat{D}^- - i\omega^-\right)f_{-1} + v\left(\mathit{sin}\,\gamma\, {\partial}/{\partial\gamma} + 2\cos\gamma\right)\left(K^+ f_2 + K^- f_{-2}\right) = R_0$$

$$\left(1 + i\mu\frac{\hat{D}_0 + il\omega_0}{\Omega + l\Omega_H}\right)f_l + \frac{i\mu}{\Omega + l\Omega_H}\left(\left[\hat{D}^- - i(l-1)\omega^-\right]f_{l-1} + \left[\hat{D}^+ + i(l+1)\omega^+\right]f_{l+1} + \right.$$

$$\left. vK^-\left[\mathit{sin}\,\gamma\frac{\partial}{\partial\gamma} - (l-2)\cos\gamma\right]f_{l-2} + vK^+\left[\mathit{sin}\,\gamma\frac{\partial}{\partial\gamma} + (l+2)\cos\gamma\right]f_{l+2}\right) = R_l \tag{1}$$

where the next notations are introduced

$$\omega_0 = \frac{v\cos\gamma}{2}\left[\left(\vec{n}\cdot\nabla\times\vec{n}\right) + \left(\vec{\tau}\cdot\nabla\times\vec{\tau}\right) - 2\left(\vec{h}\cdot\nabla\times\vec{h}\right)\right]; \quad \omega^{\pm} = \frac{1}{2}\left(\omega_1 \pm i\omega_2\right);$$

$$\omega_1 = \frac{v\cos^2\gamma}{\mathit{sin}\,\gamma}\left(\vec{n}\cdot\nabla\times\vec{h}\right) - v\,\mathit{sin}\,\gamma\left(\vec{h}\cdot\nabla\times\vec{n}\right); \quad \omega_2 = \frac{v\cos^2\gamma}{\mathit{sin}\,\gamma}\left(\vec{\tau}\cdot\nabla\times\vec{h}\right) - v\,\mathit{sin}\,\gamma\left(\vec{h}\cdot\nabla\times\vec{\tau}\right);$$

$$K^{\pm} = \frac{1}{2}\left(K_1 \pm iK_2\right); \quad K_1 = \frac{1}{2}\left[\left(\vec{\tau}\cdot\nabla\times\vec{n}\right) + \left(\vec{n}\cdot\nabla\times\vec{\tau}\right)\right]; \quad K_2 = \frac{1}{2}\left[\left(\vec{n}\cdot\nabla\times\vec{n}\right) - \left(\vec{\tau}\cdot\nabla\times\vec{\tau}\right)\right];$$

$$\hat{R}_0 = -\frac{e}{M}E_3\cos\gamma\frac{\partial F_0}{\partial v}, \quad \hat{R}_l = -\frac{e\left(E_1 - ilE_2\right)}{2M\left(\Omega + l\Omega_H\right)}\mathit{sin}\,\gamma\frac{\partial F_0}{\partial v}, \; l = \pm 1;$$

$$\hat{R}_l = 0, \quad l \geq 2; \quad F_0 = \left({N}/{\left(2\pi v_T\right)^{3/2}}\right)\exp\left(-\frac{v^2}{2v_T^2}\right)$$

The projections of electric field $\vec{E}$ on the unit base vectors $\left(\vec{n}, \vec{\tau}, \vec{h}\right)$ are marked by the indexes (1, 2, 3), respectively, and $\Omega_H = \dfrac{e\left|B_0\right|}{Mc}$ is the ion cyclotron frequency. To describe the electron component of the plasma the values of e and M should be substituted for $-e$ and m.

The next notations for differential operators in Eq.(1) are introduced

$$\hat{D}_0 = v\cos\gamma\left(\vec{h}\cdot\nabla\right) - \frac{v}{2}\left(\nabla\cdot\vec{h}\right)\mathit{sin}\,\gamma\,{\partial}/{\partial\gamma}; \quad \hat{D}^{\pm} = \hat{D}_1 \pm i\hat{D}_2$$

$$\hat{D}_1 = v\,\mathit{sin}\,\gamma\left(\vec{n}\cdot\nabla\right) - v\left(\vec{\tau}\cdot\nabla\times\vec{h}\right)\cos\gamma\,{\partial}/{\partial\gamma}; \quad \hat{D}_2 = v\,\mathit{sin}\,\gamma\left(\vec{\tau}\cdot\nabla\right) + v\left(\vec{n}\cdot\nabla\times\vec{h}\right)\cos\gamma\,{\partial}/{\partial\gamma};$$

As usually, the formal small parameter μ is inserted in the front of small quantities ρ_λ / L replaced by *one* in final results.

Solution of Vlasov equation and evaluation of dielectric tensor components

To find the solution of Eq. (1) in the second order of the parameter $\dfrac{\rho_\lambda}{L}$ we take into account only the harmonics $|l| \leq 3$ that gives us

$$f_{\pm 2} = -i\mu\left[1 - \frac{i\mu}{\Omega \pm 2\Omega_H}\left(\hat{D}_0 \pm 2i\omega_0\right)\right]\left[\mathit{sin}\,\gamma\frac{\partial}{\partial\gamma}\left(\frac{vK^{\mp}f_0}{\Omega \pm 2\Omega_H}\right) + \frac{\left(\hat{D}^{\mp} \pm 2i\omega^{\mp}\right)f_{\pm 1}}{\Omega \pm 2\Omega_H}\right] \tag{2}$$

This approximation is enough to take the bounce resonance effect in nondiagonal components of the dielectric tensor. Note that we study only electron Cherenkov dissipation which is valid in the condition $v_T \langle\langle \left|L(\Omega-\Omega_H)\right|$ then we can neglect Ω in comparison with the electron cyclotron frequency in denominators of Eqs.(1)(2). Representing the distribution function harmonics as $f_l = f_l^{(0)} + \mu\, f_l^{(1)}$ and using the definitions $4\pi i j_\alpha / \omega = \hat{\varepsilon}_{\alpha\beta}\, E_\beta$ we get the next formulae for the dielectric tensor operator components after corresponding integration in the velocity space;

$$\hat{\varepsilon}_{13}E_3 = i\frac{4\pi^2 e^2}{m\omega\,\omega_H}\int_0^\infty v^3 dv \int_0^\pi d\gamma\, sin^2\gamma\, \hat{D}_2\hat{T}\cos\gamma\frac{\partial F_0}{\partial v}E_3.$$

$$\hat{\varepsilon}_{23}E_3 = -i\frac{4\pi^2 e^2}{m\omega\,\omega_H}\int_0^\infty v^3 dv \int_0^\pi d\gamma\, sin^2\gamma\, \hat{D}_1\hat{T}\cos\gamma\, \frac{\partial F_0}{\partial v}E_3.$$

$$\hat{\varepsilon}_{33}E_3 = -i\frac{8\pi^2 e^2}{m\omega}\int_0^\infty v^3 dv \int_0^\pi d\gamma\, sin^2\gamma\, \cos\gamma\hat{T}\cos\gamma\frac{\partial F_0}{\partial v}E_3.$$

$$\hat{\varepsilon}_{11}E_1 = i\frac{2\pi^2 e^2}{m\omega\,\omega_H}\int_0^\infty v^3 dv \int_0^\pi d\gamma\, sin^2\gamma\, \hat{D}_2\hat{T}\left(\hat{D}_2+\omega_1\right)\frac{sin\gamma}{\omega_H}\frac{\partial F_0}{\partial v}E_1.$$

$$\hat{\varepsilon}_{21}E_1 = -i\frac{2\pi^2 e^2}{m\omega\,\omega_H}\int_0^\infty v^3 dv \int_0^\pi d\gamma\, sin^2\gamma\, \hat{D}_1\hat{T}\left(\hat{D}_2+\omega_1\right)\frac{sin\gamma}{\omega_H}\frac{\partial F_0}{\partial v}E_1.$$

$$\hat{\varepsilon}_{31}E_1 = -i\frac{4\pi^2 e^2}{m\omega}\int_0^\infty v^3 dv \int_0^\pi d\gamma\, sin^2\gamma\, \cos\gamma\hat{T}\left(\hat{D}_2+\omega_1\right)\frac{sin\gamma}{\omega_H}\frac{\partial F_0}{\partial v}E_1.$$

$$\hat{\varepsilon}_{12}E_2 = -i\frac{2\pi^2 e^2}{m\omega\,\omega_H}\int_0^\infty v^3 dv \int_0^\pi d\gamma\, sin^2\gamma\, \hat{D}_2\hat{T}\left(\hat{D}_1-\omega_2\right)\frac{sin\gamma}{\omega_H}\frac{\partial F_0}{\partial v}E_2.$$

$$\hat{\varepsilon}_{22}E_2 = i\frac{2\pi^2 e^2}{m\omega\,\omega_H}\int_0^\infty v^3 dv \int_0^\pi d\gamma\, sin^2\gamma\, \hat{D}_1\hat{T}\left(\hat{D}_1-\omega_2\right)\frac{sin\gamma}{\omega_H}\frac{\partial F_0}{\partial v}E_2.$$

$$\hat{\varepsilon}_{32}E_2 = i\frac{4\pi^2 e^2}{m\omega}\int_0^\infty v^3 dv \int_0^\pi d\gamma\, sin^2\gamma\, \cos\gamma\hat{T}\left(\hat{D}_1-\omega_2\right)\frac{sin\gamma}{\omega_H}\frac{\partial F_0}{\partial v}E_2.$$

In above equations the operator $\hat{T}$ is the inverse operator of the operator $\left(-i\Omega+\hat{D}_0\right)$. The parallel component of the dielectric tensor is studied in [2-4]. Here, we present the results of analysis of the imaginary part of the parallel tensor component for LART,

$$\hat{\varepsilon}^{m,m}_{33(unt.)} = \frac{\sqrt{2}\,\varepsilon\,\omega_{pe}^2}{\sqrt{\pi}\,\Omega\, k_0 v_T}\sum_r \int_{k_0}^1 \frac{dk}{k^3}\int_{-\infty}^{\infty}\frac{u^4\exp\left(\frac{-u^2}{2}\right)\left[Q^{m,r}_{unt.1}(u,k)\right]^2 du}{u\left(r+m+\frac{nq_t}{\sqrt{1-\varepsilon^2}}\right)-\frac{\Omega}{\omega_{unt.b}}-\frac{i\nu_e}{\omega_{unt.b}}} \qquad (3)$$

$$\mathrm{Im}\,\hat{\varepsilon}_{33(unt.)}^{m,m} = \frac{\sqrt{2\pi}\,\omega_{pe}^{2}\,\Omega^{3}}{\left(k_0 v_T\right)^5 k_0^2\left(2-k_0^2\right)} \sum_r \int_{k_0}^{1} k\,dk \left(\frac{2K\ A_{unt.}}{\pi}\right)^4 \frac{\left[Q_{unt.1}^{m,r}\left(V_{unt.r,m}\right)\right]^2}{\left|r+m+nq_t\right|^5} \exp\left(-\frac{V_{unt.r,m}^2}{2}\right)$$

where $V_{unt.r,m} = \dfrac{\Omega}{\omega_{unt.b}\left(r+m+n\bar{q}_t\right)}$ is the resonance value of the dimensionless velocity $\bar{q}_t = \dfrac{q_t}{\sqrt{1-\varepsilon^2}}$ is the tokamak safety factor. Some specific cases are shown for different plasmas in Fig.1.

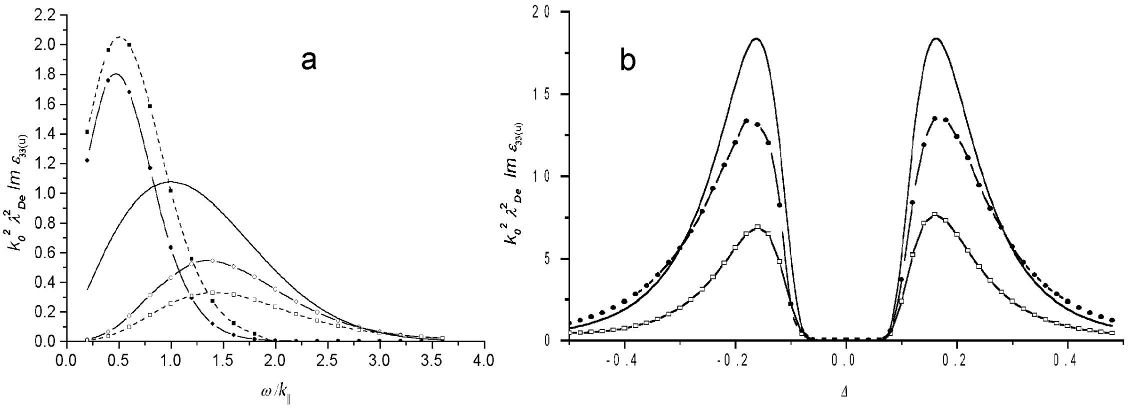

Fig.1. Plot of the imaginary part of the parallel dielectric tensor component, $k_0^2\lambda_{De}^2\ \mathrm{Im}\,\varepsilon_{33}^{mm}$, *(a) over* $\dfrac{\omega}{k_\parallel v_{Te}}$ *for trapped and untrapped electrons* $m+nq_t = 4.2$ *in comparison with cylindrical case the solid line (a), and (b) over* Δ *for untrapped electrons* $\Omega/k_0 v_T = 0.3$. *The line with black squares (a) represents trapped particle part for* $\varepsilon = \dfrac{2}{3}$, *with black circus (a) represents trapped particle part for* $\varepsilon = \dfrac{1}{2}$ *empty squares and circus(a-b) are related to* $\varepsilon = \dfrac{1}{2}$, *the line with black circus (b) is related to* $\varepsilon = 0.25$ *and the solid line (b) is calculated from Eq.(16) for ε=0.25.*

In the conclusion, we can say that the wave dissipation is strongly enhanced for waves with phase velocity larger than thermal velocity and strongly modified near rational magnetic surfaces because of strong modulation of parallel velocity of electrons. This phenomena can be effectively used for RF plasma heating, current drive and as stabilizing effect of drift instabilities at the rational magnetic surfaces in low aspect ratio tokamaks.

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